Exercise 40

Find a function f and a number a such that

$$\lim_{h \to 0} \frac{(2+h)^6 - 64}{h} = f'(a)$$

Solution

Recall the definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Comparing this with the given equation,

$$f'(a) = \lim_{h \to 0} \frac{(2+h)^6 - 2^6}{h},$$

indicates that

$$f(x) = x^6 \quad \text{and} \quad a = 2.$$